Ramaty High Energy Solar Spectroscopic Imager
(RHESSI)
Object Spectral Analysis Executive
(OSPEX)

THIN-TARGET BREMSSTRAHLUNG

IDL CODE DOCUMENTATION

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Description

This program computes the bremsstrahlung X-ray/gamma-ray spectrum from the interaction of energetic electrons with a uniform, thin-target plasma. The photon flux in photons sec^{-1} keV^{-1} cm^{-2} is computed as a function of photon energy. Angular dependence of the electron distribution function and radiation is not included. Therefore, both are assumed to be isotropic.

The program consists of the following files, each containing a single IDL procedure or function:

F_Thin2 | F_vth_Thin2 | …

… | Brm2_ThinTarget | Brm2_Dmlin | Brm_GauLeg | Brm_GauLeg53 or Brm_GauLeg54 | Brm2_Fthin | Brm_Bremcross | Brm2_Distrn

The batch file `Brm_Thin_Test.pro` can be used to test `Brm2_ThinTarget.pro` and plot the output. At the IDL command line type `@brm_thin_test`. Photon flux is plotted as a function of photon energy in keV. This runs both `brm2_thintarget.pro`, described here, and the similar, slower program, `brm_bremspec.pro`, and compares the results.

The photon flux at energy E_{ph} is computed from the equation

$$Flux = \frac{nNV}{4\pi(AU)^2}\frac{1}{(mc^2)^2}\int_{E_{ph}}^{E_{eHigh}} f(\gamma)\,\mathrm{v}\,\sigma(E_{ph},E_{el})dE_{el}\ .$$

Here n and N are the number densities (cm^{-3}) of the thermal plasma and energetic electrons, respectively. V is the volume of the emitting region and AU is one astronomical unit (in cm), taken to be the distance from the source to the detector. The mass of the electron and the speed of light are m and c, respectively, and mc^2 is the rest mass energy of the electron (in keV).

The integral is over electron energy, E_{el} (in keV). E_{eHigh} is the maximum energy of the radiating electrons. The electron density distribution function (electrons cm^{-3} keV^{-1}) is $Nf(\gamma)$, where γ is the relativistic gamma factor, related to the electron (kinetic) energy through $E_{el} = (\gamma - 1)mc^2$, and $f(\gamma)$ is normalized so that

$$\int_{\gamma_{Low}}^{\gamma_{High}} f(\gamma)\, d\gamma \;=\; \int_{E_{eLow}}^{E_{eHigh}} f(E_{el})\, dE_{el} \;=\; 1.$$

The remaining terms in the integrand are v, the electron speed, and $\sigma(E_{ph}, E_{el})$, the bremsstrahlung cross section. The cross section is from equation (4) of E. Haug, *Astron. Astrophys.* 326, 417 (1997). This cross section closely follows the relativistic cross section of Bethe and Heitler, equation 3BN in H.W. Koch and J. W. Motz, *Rev. Mod. Phys.* 31, 920 (1959), but requires fewer computations. The multiplicative Elwert correction to the Born approximation (G. Elwert, *Ann. Physik* 34, 178, 1939) is included in the cross section. The two factors of mc^2 in the denominator of the numerical coefficient are from (1) the bremsstrahlung cross section, which is differential in E_{ph}/mc^2 rather than E_{ph}, and (2) the use of $f(\gamma)$, differential in the relativistic gamma factor, γ, instead of $f(E)$.

The integration is performed by Function `Brm2_Dmlin` using Gaussian quadrature. This function and Procedures `Brm_GauLeg53` and `Brm_GauLeg54` are adapted from Press et al., *Numerical Recipes in Fortran 77*, Second Edition (Cambridge University Press, 1992). The variable RERR determines the fractional error in the integration steps.

A double-power-law electron distribution function with a high- and a low-energy cutoff is provided by Procedure `Brm2_Distrn`. To facilitate the numerical integration when the photon energy, EPH, is less than the double-power-law break energy, EEBRK, `Brm2_ThinTarget` performs the integration in two parts, above and below EEBRK (or, equivalently, GAMBRK) and sums the parts. Since the range of integration typically covers an order of magnitude or more in electron energy, with a smaller contribution to the integrals at high energies, the integrals are performed over the logarithm of the energy, giving $L = log_{10}(E)$, $E = 10^L$ and $dE = 10^L \ln(10)\, dL$. This transformation boosts the speed of the numerical integration by a factor ~2–10.

The default (EFD = 1, see Input Parameters) is to provide the electron flux density distribution function (electrons s^{-1} cm^{-2} keV^{-1}) rather than the electron density distribution function (electrons cm^{-3} keV^{-1}). N is replaced by the integrated electron flux density (electrons s^{-1} cm^{-2}) and the integral of $f \cdot v$ over E_{el} (or γ) is normalized to 1. All fit parameters A describe the electron flux density function.

Contact and Acknowledgements

The Fortran version of this program, BREMSPEC, was originally developed in 2001 by Gordon D. Holman, NASA/Goddard Space Flight Center. It can be found at

http://hesperia.gsfc.nasa.gov/hessi/modelware.htm.

Sally House, a summer student intern at Goddard Space Flight Center, developed the IDL version of this program in 2001.

Yang Su, a graduate student at PMO/CUA/GSFC, upgraded the IDL version of this program in 2009 to include the transformation of the integration over electron energy to a logarithmic energy scale.

Address questions, comments, and suggested revisions to:

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Input Parameters

Input parameters to Procedure Brm2_ThinTarget

EPH	Array of photon energies (in keV) at which to compute the photon flux.
A	Six-element array of fit parameters
P = A[1]	Power-law index of the electron (flux density or density) distribution function below EEBRK.
EEBRK = A[2]	Break energy in the electron distribution function (in keV).
Q = A[3]	Power-law index of the electron distribution function above EEBRK.
EELOW = A[4]	Low energy cutoff in the electron distribution function (in keV).
EEHIGH = A[5]	High energy cutoff in the electron distribution function (in keV).
NTHDENV = A[0]	If /EFD is specified, product of the number density of plasma ions (cm^{-3}), the flux density of nonthermal electrons (cm^{-2} s^{-1}), and the volume of the radiating source region (cm^3). In OSPEX, a[0] is normalized to 1.0d+55 cm^{-2} s^{-1}. If /EFD is not specified, product of number density of plasma ions (cm^{-3}), number density of nonthermal electrons (cm^{-3}), and volume of the radiating source region (cm^3).

Optional keyword

EFD	If /EFD is specified (i.e., EFD = 1, the default value), Procedure BRM2_DISTRN provides the (normalized) electron flux density distribution (electrons cm^{-2} s^{-1} keV^{-1}). If not specified (EFD = 0), Procedure BRM2_DISTRN provides the (normalized) electron density distribution (electrons cm^{-3} keV^{-1}).

Additional input parameters specified in Procedure Brm2_ThinTarget

Z	Mean atomic number of the target plasma, set to 1.2.
RERR	Desired relative error for evaluation of the integral. For example, RERR = 0.01 indicates that the estimate of the integral is to be correct to one digit, whereas RERR = 0.001 calls for two digits to be correct.

Output Parameters

BRM2_THINTARGET	When multiplied by A[0] * 1.0d+55, array of photon fluxes at photon energies EPH in photons s^{-1} keV^{-1} cm^{-2}. The detector is assumed to be 1 AU from the source.

Error Codes

IERGQ = IER	Array of error flags from the Gaussian quadrature integration procedure for integration over electron energies. The numerical integration is split into two parts, Part 1 below EEBRK and Part 2 above EEBRK. IER = 0 indicates convergence, IER = 1 indicates Part 1 did not converge, IER = 2 indicates Part 2 did not converge, IER = 3 indicates Part 1 & Part 2 did not converge.

Subprograms

Function F_THIN2	Calls Function F_VTH_THIN2 which calls BRM2_THINTARGET and sets the photon energies EPH to the mean of the photon energy band edges when OSPEX provides EPH as a two-dimensional array of band edges. F_VTH_THIN2 also multiplies the output of BRM2_THINTARGET by a[0] * 1.0d+55 to obtain the final, properly normalized values of the photon flux.
Function BRM2_THINTARGET	Inputs or specifies all input parameters and calls BRM2_DMLIN to compute the photon flux for all photon energies.
Function BRM2_FTHIN	Computes the integrand for the numerical integration. Calls BRM2_DISTRN and BRM_BREMCROSS. The normalization coefficient (FCOEFF) is computed in Procedure BRM2_THINTARGET.
Procedure BRM_BREMCROSS	Computes the bremsstrahlung cross section as a function of Z, incident electron energy (EEL), and radiated photon energy (EPH).
Procedure BRM2_DISTRN	Computes the normalized electron distribution function, $f(\gamma)$v (for EFD=1) or $f(\gamma)$. A double-power-law distribution function is provided.
Function BRM2_DMLIN	Performs the Gaussian quadrature numerical integration. Repeatedly doubles the number of points evaluated until convergence, specified by RERR, is obtained or the maximum number of points, specified by the parameter MAXFCN, is reached. IER = 1, 2, or 3 is returned if the integration fails to converge. MAXFCN should be less than or equal to 2^NLIM, or 4096 with NLIM = 12. The outer numerical integration is split into two parts because of the discontinuity in the electron distribution function at the break energy (E_{eBreak}).
Procedure BRM_GAULEG	Calls SINCE_VERSION and branches to either BRM_GAULEG53 or BRM_GAULEG54, depending upon the version of IDL used.

Procedure BRM_GAULEG[53 or 54]	Computes the Gaussian quadrature abscissas and weights for Function BRM2_DMLIN. Adapted from Press et al., *Numerical Recipes in Fortran 77*, Second Edition (Cambridge University Press, 1992). BRM_GAULEG53 is called when IDL Version 5.3 or less is used. BRM_GAULEG54 is called when IDL Version 5.4 or higher is used. BRM_GAULEG54 uses the IDL Function LEGENDRE, which is available in IDL 5.4 and above.
Function SINCE_VERSION	A utility in the Solar Software Tree that returns 1 if the version of IDL being used is greater than or equal to the specified version number, or 0 if less than the specified version number.