Ramaty High Energy Solar Spectroscopic Imager
(RHESSI)
Object Spectral Analysis Executive
(OSPEX)

THICK-TARGET BREMSSTRAHLUNG

VERSIONS 1 & 2

IDL CODE DOCUMENTATION

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Table of Contents

Description

This program computes the bremsstrahlung x-ray/gamma-ray spectrum from the interaction of energetic electrons with thick-target plasma. The photon flux in photons sec^{-1} keV^{-1} cm^{-2} is computed as a function of photon energy. Angular dependence of the electron distribution function and radiation is not included. Therefore, both are assumed to be isotropic.

Both versions of the program compute the photon flux spectra using the electron collisional energy loss rate and the relativistically correct Haug (1997) approximation to the Bethe-Heitler bremsstrahlung cross section. Version 1 performs a double numerical integration to obtain the photon fluxes. Either the electron flux distribution function (electrons s^{-1} keV^{-1}) or the electron density distribution function (electrons cm^{-3} keV^{-1}) can be supplied. Version 2 reverses the order of integration so that the integral over the electron flux distribution function can be supplied.

In both versions of the program the electron distribution function is taken to be a double power law with high- and low-energy cutoffs. Since in Version 2 the integral over this distribution function is obtained analytically, Version 2 is much faster than Version 1. The run speed of Version 2 is also increased by a coordinate transformation in the integration. The electron distribution function can be modified by the user in both versions of the program.

Version 1

The program consists of the following files, each containing a single IDL procedure or function:

F_Thick | F_vth_Thick | Brm_BremThick | …

| Brm_GauLeg53 or
| Brm_Gauleg | Brm_GauLeg54
… | Brm_DmlinO |
| Brm_Fthick | Brm_Distrn
| | Brm_GauLeg53 or
| Brm_DmlinI | Brm_Gauleg | Brm_GauLeg54
|
| Brm_Finner | Brm_Bremcross
| Brm_Eloss

The batch file `Brm_Thick_Test.pro` can be used to test `Brm_BremThick.pro` and plot the output. At the IDL command line type `@brm_thick_test`. Photon flux is plotted as a function of photon energy in keV. The results are compared to those from Version 2.

Following J. C. Brown, Solar Physics 18, 489 (1971), the photon flux at energy E_{ph} is computed from the equation

$$Flux = \frac{nNA}{4\pi(AU)^2}\frac{1}{(mc^2)^2}\int_{E_{ph}}^{E_{eHigh}} f(\gamma_o)\,\mathrm{v}_o\left[\int_{E_{ph}}^{E_o}\frac{\sigma(E_{ph},E)\,\mathrm{v}}{dE/dt}dE\right]dE_o\,.$$

Here n and N are the number densities (cm^{-3}) of the thermal plasma and energetic electrons, respectively. A is the area of the emitting region and AU is one astronomical unit (in cm), taken to be the distance from the source to the detector. The mass of the electron and the speed of light are m and c, respectively, and mc^2 is the rest mass energy of the electron (in keV). The two factors of mc^2 in the denominator of the coefficient come from (1) the bremsstrahlung cross section, which is differential in E_{ph}/mc^2 rather than E_{ph}, and (2) the use of $f(\gamma_0)$, differential in γ_0, instead of $f(E_0)$. In the program the integrations are over $d\gamma$ and $d\gamma_0$, giving two additional factors of mc^2 in the denominator of the numerical coefficient.

The integrals are over electron energy, E or E_o (in keV). E_{eHigh} is the maximum energy of the radiating electrons. The electron distribution function is $Nf(\gamma)$, where γ is the relativistic gamma factor, related to the electron (kinetic) energy through $E_{el} = (\gamma - 1)mc^2$, and $f(\gamma)$ is normalized so that

$$\int_{\gamma_{eLow}}^{\gamma_{eHigh}} f(\gamma)d\gamma = 1\,.$$

The remaining terms in the integrand are v, the electron speed, $\sigma(E_{ph}, E_{el})$, the bremsstrahlung cross section, and dE/dt, the single electron energy loss rate. The cross section is from equation (4) of E. Haug, *Astron. Astrophys.* 326, 417 (1997). This cross section closely follows the relativistic cross section of Bethe and Heitler, equation 3BN in H.W. Koch and J. W. Motz, *Rev. Mod. Phys.* 31, 920 (1959), but requires fewer computations. The multiplicative Elwert correction to the Born approximation (G. Elwert, *Ann. Physik* 34, 178, 1939) is included in the cross section.

The energy loss rate, *dE/dt*, is assumed to be dominated by collisional losses. Following J. Leach & V. Petrosian, *The Astrophysical Journal* 251, 781 (1981), it is taken to be

$$\frac{dE}{dt} = 4\pi r_0^2 (mc^2) nc \ln \Lambda / \beta .$$

Here r_0 is the classical radius of the electron, $\ln\Lambda$ is the Coulomb logarithm, and β is v/c. Since collisional losses are proportional to the target number density, the plasma density, *n*, cancels out of the photon flux calculation.

The integrations are performed by Functions `Brm_DmlinO` and `Brm_DmlinI` using Gaussian quadrature. These functions and Procedures `Brm_GauLeg53` and `Brm_GauLeg54` are adapted from Press et al., *Numerical Recipes in Fortran 77*, Second Edition (Cambridge University Press, 1992). The variable RERR determines the fractional error in the integration steps.

A double-power-law electron distribution function is provided by Procedure `Brm_Distrn`. To facilitate the numerical integration when the photon energy, EPH, is less than the double-power-law break energy, EEBRK, `Brm_BremThick` performs the outer integration in two parts, above and below EEBRK (or, equivalently, GAMBRK) and sums the parts. Similarly, Function `Brm_FThick` carries out the inner integration in two parts when appropriate.

Version 2

The program consists of the following files, each containing a single IDL procedure or function:

F_Thick2 | F_vth_Thick2 | …

… | Brm2_ThickTarget | Brm2_Dmlino | Brm_GauLeg | Brm_GauLeg53 or Brm_GauLeg54 | Brm2_Fouter | Brm_Bremcross | Brm_Eloss | Brm2_F_Distrn

The batch file `Brm_Thick_Test.pro` can be used to test `Brm2_ThickTarget.pro` and plot the output. At the IDL command line type `@brm_thick_test`. Photon flux is plotted as a function of photon energy in keV. The results are compared to those from Version 1.

The photon flux at photon energy ε is computed from the equation

$$Flux(\varepsilon) = \frac{n}{4\pi(AU)^2}\frac{1}{mc^2}\int_{\varepsilon}^{E_{eHigh}} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}\left[\int_{E}^{E_{eHigh}} f(E_0)dE_0\right]dE$$

$$= \frac{n}{4\pi(AU)^2}\frac{1}{mc^2}\int_{\varepsilon}^{E_{eHigh}} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}F(E)dE$$

The Procedure `Brm2_F_distrn.pro` calculates and returns the integrated electron flux distribution function, $F(E)$. The electron flux distribution function is taken to be a double power law with a high- and a low-energy cutoff. The factor of mc^2 in the denominator of the numerical coefficient is from the bremsstrahlung cross section, which is differential in ε/mc^2 rather than ε.

To avoid integrating over discontinuities and other sudden changes in the integrand, the numerical integration is split into three parts: (1) below the low-energy cutoff, (2) between the low-energy cutoff and the break energy in the double power law, and (3) above the break energy and below the high-energy cutoff. Also, since the range of integration typically covers an order of magnitude or more in electron energy, with a smaller contribution to the integrals at high energies, the integrals are performed over the logarithm of the energy, giving $L=log_{10}(E)$, $E=10^L$ and $dE=10^L \ln(10)\, dL$. This transformation further boosts the

speed of the numerical integration. Then the photon flux at photon energy ε is given by:

$$Flux(\varepsilon) = \frac{n}{4\pi(AU)^2}\ \frac{1}{mc^2}\int_{\varepsilon}^{E_{eHigh}} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}\ F(E)\,dE$$

$$= \frac{n}{4\pi(AU)^2}\ \frac{1}{mc^2}\left[\int_{\varepsilon}^{E_{eLow}} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt} F(E)\,dE + \int_{E_{eLow}}^{E_{eBreak}} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}\ F(E)\,dE + \int_{E_{eBreak}}^{E_{eHigh}} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt} F(E)\,dE\right]$$

$$= \frac{n}{4\pi(AU)^2}\ \frac{1}{mc^2}\left[\int_{\lg(\varepsilon)}^{\lg(E_{eLow})} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}\ F(E)\ 10^{L}\ln(10)\,dL + \int_{\lg(E_{eLow})}^{\lg(E_{eBreak})} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}\ F(E)\ 10^{L}\ln(10)\,dL + \int_{\lg(E_{eBreak})}^{\lg(E_{eHigh})} \frac{\sigma(\varepsilon,E)\mathrm{v}}{dE/dt}\ F(E)\ 10^{L}\ln(10)\,dL\right]$$

These changes are implemented in subroutine `Brm2_DmlinO.pro`.

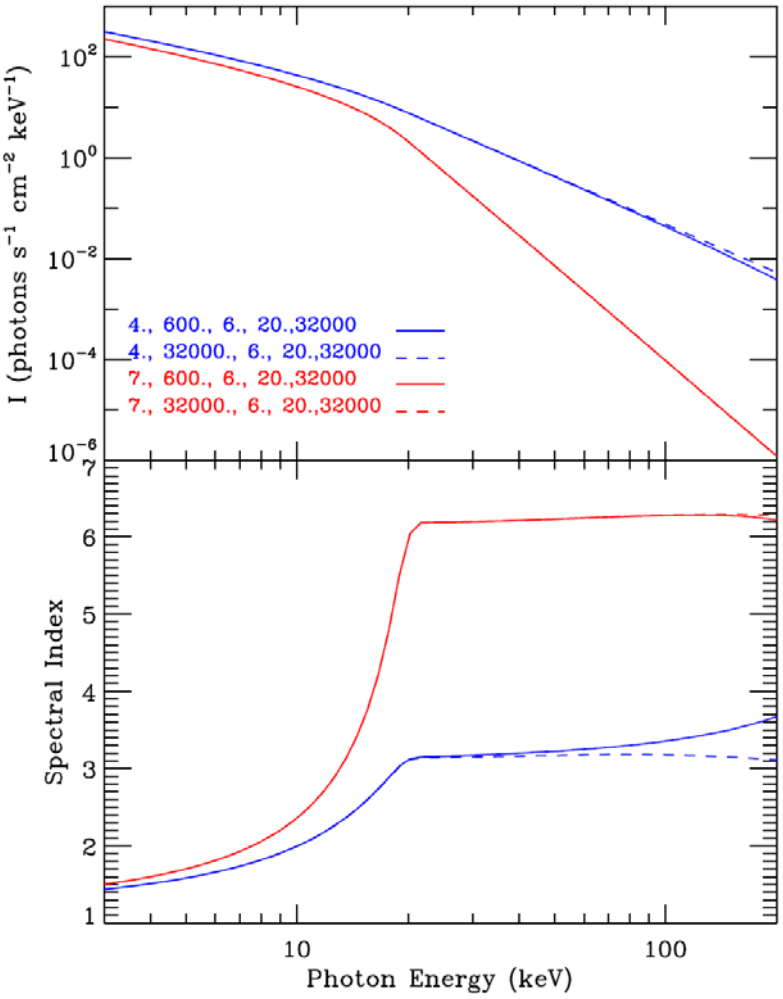
Other changes in Version 2:

1) The relative error is set to RERR = 10^{-4}. Even with this value, Version 2 is at least 10-100 times faster than Version 1 with RERR = 10^{-3}.

2) The default parameters for ThickTarget in OSPEX are changed from [1, 4, 600, 6, 10, 32000] to [1, 4, 150, 6, 20, 32000] to best represent the electron distribution for typical flare HXR spectra. To use a single power-law electron distribution during the fit, change the break energy to E_{eHigh} or greater, or to E_{eLow} or lower, and use the free mask to fix the value of E_{eBreak} (see Notes section).

3) The starting number of points for the numerical integrations, NPOINT (`Brm2_DmlinO`), is changed from 2 to 4. The value NPOINT = 2 is almost always not high enough for convergence and it may in some cases give incorrect fluxes when compared to the result for NPOINT = 4 to check for convergence within the specified tolerance RERR.

Notes



1) To use a single power-law electron distribution, change the break energy to a value that is equal to the high energy cutoff or higher (then low δ is used); or change it to a value that is equal to the low energy cutoff or lower (high δ is used). You also need to change the free mask in OSPEX for both the break energy and high δ or low δ to 0.

Two examples of how a break in the spectrum well above the energies of interest can affect the spectrum at lower energies are shown in the image on this page. The image shows how electrons above 600 keV can contribute significantly to the photon flux below 200 keV. For both cases the difference in flux and spectral index between the spectrum from a single power-law electron distribution and that from a double power-law distribution is shown. The solid curve shows the spectrum for a break in the electron distribution at 600 keV, while the dashed curve is from the same electron distribution without the break. The differences below 200 keV are especially significant for the downward break of two in spectral index (solid blue curve). The impact on the spectrum at the lower energies can be even greater for smaller power-law indices (flatter spectra). (For examples of how the value of the high-energy cutoff affects photon spectra, see Figures 9 & 10 of Holman, 2003, ApJ, 586, 606.)

2) If you want to plot the derivative of the flux or the spectral index of the photon spectrum as a function of photon energy, you should set RERR in `Brm2_ThickTarget.pro` to 1.d-6, because the spectral index is more sensitive than the flux to the value of RERR. Then (for Version 2):

FLUX = `F_Thick2`(EPH, A) or FLUX = `Brm2_ThickTarget`(EPH, A)

INDEX = –`Deriv`(`Alog10`(EPH), `Alog10`(FLUX)).

Contact and Acknowledgements

The Fortran version of this program, BREMTHICK, was originally developed in 2001 by Gordon D. Holman, NASA/Goddard Space Flight Center. It can be found at

http://hesperia.gsfc.nasa.gov/hessi/modelware.htm.

Sally House, a summer student intern at Goddard Space Flight Center, developed the IDL version of this program in 2001.

Yang Su, a graduate student at PMO/CUA/GSFC, upgraded the IDL version of this program to Version 2 in 2009.

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Input Parameters

Input parameters to Function `Brm_BremThick` *or* `Brm2_BremThick`

EPH	Array of photon energies (in keV) at which to compute the photon flux.
A	Six-element array of fit parameters
P = A[1]	Power-law index of the electron distribution function below EEBRK.
EEBRK = A[2]	Break energy in the electron distribution function (in keV).
Q = A[3]	Power-law index of the electron distribution function above EEBRK.
EELOW = A[4]	Low energy cutoff in the electron distribution function (in keV).
EEHIGH = A[5]	High energy cutoff in the electron distribution function (in keV).
DENA = A[0]	Product of the total flux density of nonthermal electrons (cm^{-2} s^{-1}) and the area of the radiating source region (cm^2), i.e., the total electron flux in electrons s^{-1} (Version 1: EFD = 1). In OSPEX, a[0] is normalized to 1.d+35 electrons s^{-1}. Version 1, EFD = 0: Product of the number density of nonthermal electrons (cm^{-3}) and the area of the radiating source region (cm^2).

Additional input parameters specified in Procedure BRM_BREMTHICK

Z	Mean atomic number of the target plasma, set to 1.2.
RERR	Desired relative error for evaluation of the integral. For example, RERR = 0.01 indicates that the estimate of the integral is to be correct to one digit, whereas RERR = 0.001 calls for two digits to be correct.

EFD (Version 1)	If true (EFD = 1), Procedure `Brm_Distrn` provides the (normalized) electron flux distribution (electrons s^{-1} keV^{-1}). EFD = 1 is the default value. If false (EFD = 0), Procedure `Brm_Distrn` provides the (normalized) electron density distribution (electrons cm^{-3} keV^{-1}).

Output Parameters

BRM_BREMTHICK	When multiplied by A[0] * 1.0d+35, array of photon fluxes at photon energies EPH in photons s^{-1} keV^{-1} cm^{-2}. The detector is assumed to be 1 AU from the source.

Error Codes

IERGQL	Array of error flags from the outer Gaussian quadrature integration procedure for integration over electron energies below EEBRK. The numerical integration converged where IERGQL = 0. It did not converge where IERGQL = 120.
IERGQH	Array of error flags from the outer Gaussian quadrature integration procedure for integration over electron energies above EEBRK. The numerical integration converged where IERGQH = 0. It did not converge where IERGQH = 120.
IERGQI	Array of error flags from the inner Gaussian quadrature integrations. The numerical integrations converged where IERGQI = 0. Elsewhere, the value of IERGQI is the number of integrations that did not converge.

Version 1 Subprograms

Function F_THICK	Calls Function F_VTH_THICK which calls BRM_BREMTHICK and sets the photon energies EPH to the mean of the photon energy band edges when OSPEX provides EPH as a two-dimensional array of band edges. F_VTH_THICK also multiplies the output of BRM_BREMTHICK by a[0] * 1.0d+35 to obtain the final, properly normalized values of the photon flux.
Function BRM_BREMTHICK	Inputs or specifies all input parameters and calls BRM_DMLINO to compute the photon flux for all photon energies. The outer numerical integration is split into two parts because of the discontinuity in the electron distribution function at the break energy (E_{eBreak}).
Procedure BRM_ELOSS	Computes the energy dependent terms of the energy loss rate, returned as DEDT. The numerical coefficient, DECOEFF, is computed in Procedure BRM_BREMTHICK.
Function BRM_FINNER	Computes the integrand for the inner numerical integration. Calls BRM_BREMCROSS and BRM_ELOSS.
Function BRM_FTHICK	Computes the integrand for the outer numerical integration. Calls BRM_DMLINI and BRM_FINNER to compute the inner integral. Calls BRM_DISTRN. The normalization coefficient, FCOEFF, is computed in Procedure BRM_BREMTHICK. Like the outer integration (see BRM_BREMTHICK), the inner integration is split into two parts.
Procedure BRM_BREMCROSS	Computes the bremsstrahlung cross section as a function of Z, incident electron energy (EEL), and radiated photon energy (EPH).
Procedure BRM_DISTRN	Computes the normalized electron distribution function, $f(\gamma)$. A double-power-law distribution function is provided.
Function BRM_DMLIN[O or I]	Performs the Gaussian quadrature numerical integrations. Repeatedly doubles the number of

	points evaluated until convergence, specified by RERR, is obtained or the maximum number of points, specified by the parameter MAXFCN, is reached. For DMLINO, IER = 120 is returned if the integration fails to converge. For DMLINI, IER = 1 is returned if the integration fails to converge. MAXFCN should be less than or equal to 2^NLIM, or 4096 with NLIM = 12.
Procedure BRM_GAULEG	Calls SINCE_VERSION and branches to either BRM_GAULEG53 or BRM_GAULEG54, depending upon the version of IDL used.
Procedure BRM_GAULEG[53 or 54]	Computes the Gaussian quadrature abscissas and weights for Function BRM_DMLIN[O or I]. Adapted from Press et al., *Numerical Recipes in Fortran 77*, Second Edition (Cambridge University Press, 1992). BRM_GAULEG53 is called when IDL Version 5.3 or less is used. BRM_GAULEG54 is called when IDL Version 5.4 or higher is used. BRM_GAULEG54 uses the IDL function LEGENDRE, which is available in IDL 5.4 and above.
Function SINCE_VERSION	A utility in the Solar Software Tree that returns 1 if the version of IDL being used is greater than or equal to the specified version number, or 0 if less than the specified version number.

Version 2 Subprograms

Function F_THICK2	Calls Function F_VTH_THICK2 which calls BRM2_THICKTARGET and sets the photon energies EPH to the mean of the photon energy band edges when OSPEX provides EPH as a two-dimensional array of band edges. F_VTH_THICK2 also multiplies the output of BRM_BREMTHICK by a[0] * 1.0d+35 to obtain the final, properly normalized values of the photon flux.
Function BRM2_THICKTARGET	Inputs or specifies all input parameters and calls BRM_DMLINO to compute the photon flux for all photon energies.
Procedure BRM_ELOSS	[Same as Version 1] Computes the energy dependent terms of the energy loss rate, returned as DEDT. The numerical coefficient, DECOEFF, is computed in Procedure BRM2_THICKTARGET.
Function BRM2_FOUTER	Computes the integrand for the outer numerical integration. Calls BRM_BREMCROSS and BRM_ELOSS. The normalization coefficient, FCOEFF, is computed in Procedure BRM2_THICKTARGET.
Procedure BRM_BREMCROSS	[Same as Version 1] Computes the bremsstrahlung cross section as a function of Z, incident electron energy (EEL), and radiated photon energy (EPH).
Procedure BRM2_F_DISTRN	Calculates and returns the integrated electron flux distribution function, F(E). The electron flux distribution function is taken to be a double power law with a high- and a low-energy cutoff.
Function BRM2_DMLINO	Performs the Gaussian quadrature numerical integrations. Repeatedly doubles the number of points evaluated until convergence, specified by RERR, is obtained or the maximum number of points, specified by the parameter MAXFCN, is reached. IER = 1 is returned if the integration fails to converge. MAXFCN should be less than or equal to 2^NLIM, or 4096 with NLIM = 12. The numerical integration is split into three parts: (1)

	below the low-energy cutoff, (2) between the low-energy cutoff and the break energy, and (3) above the break energy and below the high-energy cutoff.
Procedure BRM_GAULEG	[Same as Version 1] Calls SINCE_VERSION and branches to either BRM_GAULEG53 or BRM_GAULEG54, depending upon the version of IDL used.
Procedure BRM_GAULEG[53 or 54]	[Same as Version 1] Computes the Gaussian quadrature abscissas and weights for Function BRM2_DMLINO. Adapted from Press et al., *Numerical Recipes in Fortran 77*, Second Edition (Cambridge University Press, 1992). BRM_GAULEG53 is called when IDL Version 5.3 or less is used. BRM_GAULEG54 is called when IDL Version 5.4 or higher is used. BRM_GAULEG54 uses the IDL function LEGENDRE, which is available in IDL 5.4 and above.
Function SINCE_VERSION	[Same as Version 1] A utility in the Solar Software Tree that returns 1 if the version of IDL being used is greater than or equal to the specified version number, or 0 if less than the specified version number.