Built-in analytical electron distributions

The built-in analytical electron distribution functions, as a rule (see below) are assumed to have the factorized form:

$$f(E,\mu) = u(E)g(\mu), \tag{1}$$

where E is the electron energy, $\mu = \cos\alpha$, and α is the electron pitch-angle. The functions $u(E)$ and $g(\mu)$ satisfy the normalization conditions

$$2\pi \int\limits_{E_{\rm min}}^{E_{\rm max}} u(E){\rm d}E = n_{\rm e/b}, \qquad \int\limits_{-1}^{1} g(\mu){\rm d}\mu = 1, \tag{2}$$

where $n_{\rm e/b}$ is the concentration of either thermal or non-thermal electrons (depending on the distribution type). In a given volume element, the energy distribution $u(E)$ is specified by the parameter `Parms[6]`, and the pitch-angle distribution $g(\mu)$ is specified by the parameter `Parms[14]` (see the separate document `CallingConventions.pdf`).

Currently, the following energy distributions $u(E)$ are supported:

1. FFF (index = 0 or 1): free-free only *(always isotropic)*.
2. THM (index = 2): thermal.
3. PLW (index = 3): power-law over energy.
4. DPL (index = 4): double power-law over energy.
5. TNT (index = 5): continuous thermal-nonthermal (power-law) over energy.
6. KAP (index = 6): kappa-distribution.
7. PLP (index = 7): power-law over momentum.
8. PLG (index = 8): power-law over relativistic factor.
9. TNP (index = 9): continuous thermal-nonthermal (power-law) over momentum.
10. TNG (index = 10): continuous thermal-nonthermal (power-law) over relativistic factor.

In addition, there are two partially factorized energy distributions — combinations of an isotropic thermal component and a possibly anisotropic nonthermal component (the anisotropy factor $g(\mu)$ is applied to the nonthermal component only):

11. TPL (index = 11): isotropic thermal + power-law over energy.
12. TDP (index = 12): isotropic thermal + double power-law over energy.

Currently, the following pitch-angle distributions $g(\mu)$ are supported:

1. ISO (index = 0 or 1): isotropic.
2. ELC (index = 2): exponential (on the pitch-angle cosine) loss-cone.
3. GAU (index = 3): gaussian (on the pitch-angle cosine) loss-cone.
4. GAB (index = 4): directed gaussian (on the pitch-angle cosine) beam.
5. SGA (index = 5): directed super-gaussian (on the pitch-angle cosine) beam.

1 Energy distributions

Free-free only (FFF; index 0 or 1)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see below)*, in cm^{-3}.
- `Parms[6]` $= 0$ or 1.
- `Parms[18]` $= n_\mathrm{p}$ is the proton concentration *(used only as a switch, see below)*, in cm^{-3}.
- `Parms[19]` $= n_\mathrm{HI}$ is the neutral hydrogen concentration *(see below)*, in cm^{-3}.
- `Parms[20]` $= n_\mathrm{HeI}$ is the neutral helium concentration *(see below)*, in cm^{-3}.
- `Parms[22]` is the element abundance model *(see below)*.

If this index is selected, only the free-free emission from thermal plasma with isotropic Maxwellian distribution is computed; the pitch-angle distribution index `Parms[14]` has no effect. The free-free emission and absorption processes include contributions from electron-ion and electron-neutral (neutral hydrogen and helium) collisions, as described in the paper of Fleishman et al. (2021). For the electron-ion collisions, the key `Parms[22]` specifies the used element abundance model: the solar coronal abundance by Feldman (1992) for `Parms[22]=0` or the solar photospheric abundance by Scott et al. (2015) for `Parms[22]=1`; the value of `Parms[22]=-1` means that classical approximate formulae from Dulk (1985) are used.

The concentrations of the plasma components n_e (the thermal electron concentration), n_HI (the neutral hydrogen concentration) and n_HeI (the neutral helium concentration) are computed following the algorithm presented in the separate document `Diagram.pdf`, i.e.:

- If $T_0 \geq 10^5$ K then the input parameter `Parms[2]` $= n_0$ is assumed to specify the electron concentration, i.e. $n_\mathrm{e} = n_0 =$ `Parms[2]`. Other concentrations are also taken directly from the input array: $n_\mathrm{HI} =$ `Parms[19]` and $n_\mathrm{HeI} =$ `Parms[20]`.
- If $T_0 < 10^5$ K then:
  - If either $n_\mathrm{p} \neq 0$ or $n_\mathrm{HI} \neq 0$ (`Parms[18]` $\neq 0$ or `Parms[19]` $\neq 0$) then, like in the previous case, $n_\mathrm{e} = n_0 =$ `Parms[2]`, $n_\mathrm{HI} =$ `Parms[19]`, and $n_\mathrm{HeI} =$ `Parms[20]`.
  - If both $n_\mathrm{p} = 0$ and $n_\mathrm{HI} = 0$ (`Parms[18]` $= 0$ and `Parms[19]` $= 0$) then the input parameter `Parms[2]` $= n_0$ is assumed to specify the total concentration of atoms n_atom (both ionized and neutral) in the plasma. Then, the concentrations of thermal electrons n_e and neutral atoms n_HI and n_HeI are computed using this value of n_atom, the plasma temperature T_0, and Saha equation (the user-supplied value of n_HeI is overridden).

Note: By default, the free-free contribution of isotropic thermal plasma is always computed, in addition to the gyrosynchrotron emission from an analytical or/and numerically defined electron distribution. To remove this contribution (if you need to analyze the gyrosynchrotron emission only), set the key `Parms[5]` to 2 (to switch off the electron-ion collisions) or 4 (to switch off the electron-neutral collisions) or 2 + 4 (to switch off both the electron-ion and electron-neutral collisions).

Thermal distribution (THM; index 2)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 2$.
- Optionally, for low temperatures: `Parms[18]` $= n_\mathrm{p}$ is the proton concentration and `Parms[19]` $= n_\mathrm{HI}$ is the neutral hydrogen concentration, in cm^{-3}. *These parameters are used only as switches, see comments for the FFF model.*

Relativistic thermal distribution is given by the expression

$$u_\mathrm{THM}(\Gamma)\,\mathrm{d}\Gamma = \frac{n_\mathrm{e}}{2\pi}\frac{\Gamma\sqrt{\Gamma^2-1}}{\theta K_2(1/\theta)}\exp\left(-\frac{\Gamma}{\theta}\right)\,\mathrm{d}\Gamma, \tag{3}$$

where n_e is the thermal electron concentration, Γ is the Lorentz-factor, $\theta = k_\mathrm{B}T_0/(mc^2)$ is the normalized thermal energy for the temperature T_0, k_B is the Boltzmann constant, and K_2 is the MacDonald function of the second order. The thermal electron concentration n_e is computed following the algorithm presented in the separate document `Diagram.pdf` (see also comments for the FFF model).

Note: Although the background thermal plasma is present in most cases, its gyrosynchrotron emission is computed only if the user explicitly selects the thermal (or a thermal/nonthermal, see below) electron distribution in the list of parameters.

Single power-law distribution over kinetic energy (PLW; index 3)

Used parameters:

- `Parms[6]` $= 3$.
- `Parms[7]` $= n_\mathrm{b}$ is the concentration of nonthermal electrons, in cm^{-3}.
- `Parms[9]` $= E_\mathrm{min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `Parms[10]` $= E_\mathrm{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.

Power-law distributions of the nonthermal electrons over kinetic energy $E = mc^2(\Gamma-1)$ are widely used for interpretation of solar radio and hard X-ray emissions. These distributions are given by the expression

$$u_\mathrm{PLW}(E)\,\mathrm{d}E = AE^{-\delta}\,\mathrm{d}E, \text{ for } E_\mathrm{min} < E < E_\mathrm{max}, \tag{4}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_\mathrm{b}}{2\pi}\frac{\delta-1}{E_\mathrm{min}^{1-\delta}-E_\mathrm{max}^{1-\delta}}, \tag{5}$$

where n_b is the concentration of the nonthermal electrons. The logarithmic normalization for $\delta = 1$ is not implemented; however, one can arbitrarily approach this case taking δ very close but slightly different from 1.

Double power-law distribution over energy (DPL; index 4)

Used parameters:

- `Parms[6]` $= 4$.
- `Parms[7]` $= n_\mathrm{b}$ is the concentration of nonthermal electrons, in cm^{-3}.
- `Parms[9]` $= E_\mathrm{min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `Parms[10]` $= E_\mathrm{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[11]` $= E_\mathrm{break}$ is the break energy, in MeV ($E_\mathrm{min} < E_\mathrm{break} < E_\mathrm{max}$).
- `Parms[12]` $= \delta_1$ is the low-energy power-law index.
- `Parms[13]` $= \delta_2$ is the high-energy power-law index.

In this case, the electron spectrum consists of two parts (high-energy and low-energy), where both the high-energy and low-energy parts are described by power laws, but with different indices. This distribution (double power-law or broken power-law) can be described by the following expression:

$$u_\mathrm{DPL}(E)\,\mathrm{d}E = \mathrm{d}E \begin{cases} A_1 E^{-\delta_1}, & \text{for } E_\mathrm{min} < E < E_\mathrm{break}, \\ A_2 E^{-\delta_2}, & \text{for } E_\mathrm{break} \le E < E_\mathrm{max}, \end{cases} \tag{6}$$

and 0 outside the range from E_min to E_max. In the above expression, $A_1 E_\mathrm{break}^{-\delta_1} = A_2 E_\mathrm{break}^{-\delta_2}$ (to make the function continuous), $\delta_1 \neq 1$, and $\delta_2 \neq 1$. The normalization factor is given by

$$A_1^{-1} = \frac{2\pi}{n_\mathrm{b}} \left(\frac{E_\mathrm{min}^{1-\delta_1} - E_\mathrm{break}^{1-\delta_1}}{\delta_1 - 1} + E_\mathrm{break}^{\delta_2-\delta_1} \frac{E_\mathrm{break}^{1-\delta_2} - E_\mathrm{max}^{1-\delta_2}}{\delta_2 - 1} \right), \tag{7}$$

i.e., n_b is the total concentration of nonthermal electrons between E_min and E_max, and A_2 is found using the above continuity condition.

Thermal/nonthermal distribution over energy (TNT; index 5)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 5$.
- `Parms[8]` $= \varepsilon$ is the matching parameter ε.
- `Parms[10]` $= E_\mathrm{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.
- Optionally, for low temperatures: `Parms[18]` $= n_\mathrm{p}$ is the proton concentration and `Parms[19]` $= n_\mathrm{HI}$ is the neutral hydrogen concentration, in cm^{-3}. *These parameters are used only as switches, see comments for the FFF model.*

This distribution looks like a thermal distribution (THM) at low energies and a single power-law nonthermal distribution (PLW) at high energies, with a continuous transition at some energy $E_{\rm cr}$, i.e.

$$u_{\rm TNT}(E)\,{\rm d}E = {\rm d}E \begin{cases} u_{\rm THM}(E), & \text{for } E < E_{\rm cr}, \\ AE^{-\delta}, & \text{for } E_{\rm cr} \le E < E_{\rm max}, \end{cases} \tag{8}$$

and 0 for $E > E_{\rm max}$. In the above expression, $u_{\rm THM}(E)$ is the thermal distribution function (3), $A = u_{\rm THM}(E_{\rm cr})E_{\rm cr}^{\delta}$ to make the function continuous, the matching point $E_{\rm cr}$ satisfies the condition $E_{\rm cr} < E_{\rm max}$, and δ is the spectral index describing the nonthermal component. The matching point $E_{\rm cr}$ is defined as the energy corresponding to the momentum $p_{\rm cr}$

$$p_{\rm cr}^2 = \frac{p_{\rm THM}^2}{\varepsilon}, \tag{9}$$

where $p_{\rm THM}$ is the mean thermal momentum corresponding to the energy $k_{\rm B}T_0$, and the parameter ε specifies location of the turning point; the distribution becomes purely thermal when $\varepsilon < p_{\rm THM}^2/p^2(E_{\rm max})$.

The normalization condition is assumed to be the same as for the purely thermal distribution (3), which is valid for $\varepsilon \ll 1$. The thermal electron concentration $n_{\rm e}$ is computed following the algorithm presented in the separate document `Diagram.pdf` (see also comments for the FFF model); the nonthermal electron concentration $n_{\rm b}$ is computed using the above-mentioned continuity condition, and the total electron number density equals $n_{\rm e} + n_{\rm b}$.

Kappa distribution (KAP; index 6)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 6$.
- `Parms[8]` $= \varkappa$ is the parameter $\varkappa$.
- `Parms[10]` $= E_{\rm max}$ is the high-energy cutoff of the electrons, in MeV.
- Optionally, for low temperatures: `Parms[18]` $= n_{\rm p}$ is the proton concentration and `Parms[19]` $= n_{\rm HI}$ is the neutral hydrogen concentration, in cm^{-3}. *These parameters are used only as switches, see comments for the FFF model.*

Another way of describing the smooth transition from the thermal distribution to a nonthermal tail is a so-called kappa distribution, which is widely used to quantify particle distributions in the interplanetary plasma. It is convenient to express the kappa distribution in terms of the Lorentz-factor Γ:

$$u_{\rm KAP}(\Gamma)\,{\rm d}\Gamma = A\frac{\Gamma\sqrt{\Gamma^2-1}}{\theta^{3/2}\left[1+\dfrac{\Gamma-1}{(\varkappa-3/2)\theta}\right]^{\varkappa+1}}\,{\rm d}\Gamma \text{ for } E < E_{\rm max}, \tag{10}$$

and 0 otherwise. In the above expression, $\theta = k_{\rm B}T_0/(mc^2)$ is the normalized thermal energy for the temperature T_0, and $\varkappa$ is the distribution parameter ($\varkappa > 3/2$). The normalization factor A is calculated numerically by using the normalization condition (2) to provide the total

electron concentration equal to n_e; in turn, the thermal electron concentration n_e is computed following the algorithm presented in the separate document `Diagram.pdf` (see also comments for the FFF model). The nonthermal tail is more pronounced for smaller values of $\varkappa$; kappa distribution becomes purely thermal distribution when $\varkappa \to \infty$.

Note: If kappa distribution is selected, the free-free contribution (electron-ion collisions) is also computed using the formulae for isotropic kappa distribution (Fleishman & Kuznetsov 2014).

Power-law distribution over momentum (PLP; index 7)

Used parameters:

- `Parms[6]` $= 7$.
- `Parms[7]` $= n_b$ is the concentration of nonthermal electrons, in cm^{-3}.
- `Parms[9]` $= E_{min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `Parms[10]` $= E_{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.

Power-law distribution of the nonthermal electrons over the absolute value of momentum is given by the expression

$$u_{PLP}(p)\,dp = Ap^{-\delta}\,dp, \text{ for } p_{min} < p < p_{max}, \tag{11}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_b}{2\pi}\frac{\delta - 3}{p_{min}^{3-\delta} - p_{max}^{3-\delta}}, \tag{12}$$

where n_b is the concentration of nonthermal electrons, $p_{min} = p(E_{min})$, and $p_{max} = p(E_{max})$; the case of $\delta = 3$ is not implemented.

Power-law distribution over Lorentz factor (PLG; index 8)

Used parameters:

- `Parms[6]` $= 8$.
- `Parms[7]` $= n_b$ is the concentration of nonthermal electrons, in cm^{-3}.
- `Parms[9]` $= E_{min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `Parms[10]` $= E_{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.

Power-law distribution of the nonthermal electrons over Lorentz factor is given by the expression

$$u_{PLG}(\Gamma)\,d\Gamma = A\Gamma^{-\delta}\,d\Gamma, \text{ for } \Gamma_{min} < \Gamma < \Gamma_{max}, \tag{13}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_b}{2\pi}\frac{\delta - 1}{\Gamma_{min}^{1-\delta} - \Gamma_{max}^{1-\delta}}, \tag{14}$$

where n_b is the concentration of nonthermal electrons, $\Gamma_{min} = \Gamma(E_{min})$, and $\Gamma_{max} = \Gamma(E_{max})$; the case of $\delta = 1$ is not implemented.

Thermal/nonthermal distribution over momentum (TNP; index 9)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 9$.
- `Parms[8]` $= \varepsilon$ is the matching parameter ε.
- `Parms[10]` $= E_{\max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.
- Optionally, for low temperatures: `Parms[18]` $= n_{\mathrm{p}}$ is the proton concentration and `Parms[19]` $= n_{\mathrm{HI}}$ is the neutral hydrogen concentration, in cm^{-3}. *These parameters are used only as switches, see comments for the FFF model.*

This distribution is similar to the thermal/nonthermal distribution over energy (TNT) with the only difference that the nonthermal part (at $E > E_{\mathrm{cr}}$) is described by the power-law distribution over the absolute value of momentum (PLP), that is

$$u_{\mathrm{TNP}}(p)\,\mathrm{d}p = \mathrm{d}p \begin{cases} u_{\mathrm{THM}}(p), & \text{for } p < p_{\mathrm{cr}}, \\ Ap^{-\delta}, & \text{for } p_{\mathrm{cr}} \le p < p_{\max}, \end{cases} \tag{15}$$

and 0 for $p > p_{\max}$. In the above expression, $u_{\mathrm{THM}}(p)$ is the thermal distribution function (3) expressed via momentum, p_{cr} is given by Eq. (9), $p_{\max} = p(E_{\max})$. Location of the matching point and the matching and normalization conditions are the same as for the TNT distribution.

Thermal/nonthermal distribution over Lorentz factor (TNG; index 10)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 10$.
- `Parms[8]` $= \varepsilon$ is the matching parameter ε.
- `Parms[10]` $= E_{\max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.

This distribution is similar to the thermal/nonthermal distribution over energy (TNT) with the only difference that the nonthermal part (at $E > E_{\mathrm{cr}}$) is described by the power-law distribution over the Lorentz factor (PLG), that is

$$u_{\mathrm{TNG}}(\Gamma)\,\mathrm{d}\Gamma = \mathrm{d}\Gamma \begin{cases} u_{\mathrm{THM}}(\Gamma), & \text{for } \Gamma < \Gamma_{\mathrm{cr}}, \\ A\Gamma^{-\delta}, & \text{for } \Gamma_{\mathrm{cr}} \le \Gamma < \Gamma_{\max}, \end{cases} \tag{16}$$

and 0 for $\Gamma > \Gamma_{\max}$. In the above expression, $u_{\mathrm{THM}}(\Gamma)$ is the thermal distribution function (3) expressed via Lorentz factor, $\Gamma_{\mathrm{cr}} = \Gamma(p_{\mathrm{cr}})$, p_{cr} is given by Eq. (9), $\Gamma_{\max} = \Gamma(E_{\max})$. Location of the matching point and the matching and normalization conditions are the same as for the TNT distribution.

Isotropic thermal + power-law over energy (TPL; index 11)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 11$.
- `Parms[7]` $= n_\mathrm{b}$ is the concentration of nonthermal electrons, in cm^{-3}.
- `Parms[9]` $= E_\mathrm{min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `Parms[10]` $= E_\mathrm{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `Parms[12]` $= \delta$ is the power-law index.
- Optionally, for low temperatures: `Parms[18]` $= n_\mathrm{p}$ is the proton concentration and `Parms[19]` $= n_\mathrm{HI}$ is the neutral hydrogen concentration, in cm^{-3}. *These parameters are used only as switches, see comments for the FFF model.*

If this index is selected, the electron distribution function represents a sum of an isotropic thermal distribution and a (possibly anisotropic) single power-law distribution, i.e.

$$f_\mathrm{TPL}(E,\mu) = f_\mathrm{THM}(E) + u_\mathrm{PLW}(E)g(\mu), \tag{17}$$

where $f_\mathrm{THM}(E)$ is an isotropic distribution function with the energy dependence given by Eq. (3), $u_\mathrm{PLW}(E)$ is the single power-law energy distribution (4), and $g(\mu)$ is a pitch-angle distribution specified separately by the parameter `Parms[14]` (i.e., the anisotropy factor is applied only to the nonthermal component). The thermal and nonthermal components are normalized independently, to provide the electron concentrations of n_e and n_b, respectively; the thermal electron concentration n_e is computed following the algorithm presented in the separate document `Diagram.pdf` (see also comments for the FFF model), and the total electron concentration equals $n_\mathrm{e} + n_\mathrm{b}$.

Note: In contrast to the above mentioned thermal/nonthermal distributions, the TPL distribution is not made to be continuous; the thermal and nonthermal components can overlap in some range of energies. The gyrosynchrotron emissivities and absorption coefficients are computed separately for each component and then added together.

Isotropic thermal + double power-law over energy (TDP; index 12)

Used parameters:

- `Parms[1]` $= T_0$ is the plasma temperature, in K.
- `Parms[2]` $= n_0$ is either the thermal electron concentration or the total atomic concentration *(see comments for the FFF model)*, in cm^{-3}.
- `Parms[6]` $= 12$.
- `Parms[7]` $= n_\mathrm{b}$ is the concentration of nonthermal electrons, in cm^{-3}.
- `Parms[9]` $= E_\mathrm{min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `Parms[10]` $= E_\mathrm{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.

- `Parms[11]` $= E_{\rm break}$ is the break energy, in MeV ($E_{\rm min} < E_{\rm break} < E_{\rm max}$).
- `Parms[12]` $= \delta_1$ is the low-energy power-law index.
- `Parms[13]` $= \delta_2$ is the high-energy power-law index.
- Optionally, for low temperatures: `Parms[18]` $= n_{\rm p}$ is the proton concentration and `Parms[19]` $= n_{\rm HI}$ is the neutral hydrogen concentration, in cm^{-3}. *These parameters are used only as switches, see comments for the FFF model.*

This electron distribution is similar to the previous TPL distribution, but the (possibly anisotropic) nonthermal component has the double power-law energy dependence, i.e.

$$f_{\rm TDP}(E,\mu) = f_{\rm THM}(E) + u_{\rm DPL}(E) g(\mu), \tag{18}$$

where $f_{\rm THM}(E)$ is an isotropic distribution function with the energy dependence given by Eq. (3), $u_{\rm DPL}(E)$ is the double (or broken) power-law energy distribution (6), and $g(\mu)$ is a pitch-angle distribution specified separately by the parameter `Parms[14]`.

If the energy distribution index differs from the above values ($0-12$) then the free-free only model (index 0) will be used by default.

2 Pitch-angle distributions

Isotropic distribution (ISO; index 0 or 1)

Used parameters:

- `Parms[14]` $= 0$ or 1.

In this case, the electron distribution does not depend on pitch-angle, that is

$$g_{\rm ISO}(\mu) = {\rm const} = \frac{1}{2}. \tag{19}$$

Exponential loss-cone distribution (ELC; index 2)

Used parameters:

- `Parms[14]` $= 2$.
- `Parms[15]` $= \alpha_{\rm c}$ is the loss-cone boundary, in degrees.
- `Parms[16]` $= \Delta\mu$ is the loss-cone boundary width.

Symmetric loss-cone distribution with exponential boundary is given by the expression

$$g_{\rm ELC}(\mu) = A \begin{cases} 1, & \text{for } |\mu| < \mu_{\rm c}, \\ \exp\left(-\dfrac{|\mu| - \mu_{\rm c}}{\Delta\mu}\right), & \text{for } |\mu| \geq \mu_{\rm c}, \end{cases} \tag{20}$$

where $\mu_{\rm c} = \cos\alpha_{\rm c} > 0$ is the loss-cone boundary, and the parameter $\Delta\mu$ determines the sharpness of the loss-cone boundary. The normalization factor A is given by

$$A^{-1} = 2\left[\mu_{\rm c} + \Delta\mu - \Delta\mu \exp\left(\frac{\mu_{\rm c} - 1}{\Delta\mu}\right)\right]. \tag{21}$$

Gaussian loss-cone distribution (GLC; index 3)

Used parameters:

- `Parms[14]` $= 3$.
- `Parms[15]` $= \alpha_c$ is the loss-cone boundary, in degrees.
- `Parms[16]` $= \Delta\mu$ is the loss-cone boundary width.

Symmetric loss-cone distribution with gaussian boundary is given by the expression

$$g_{\mathrm{GLC}}(\mu) = A \begin{cases} 1, & \text{for } |\mu| < \mu_c, \\ \exp\left[-\dfrac{(|\mu| - \mu_c)^2}{\Delta\mu^2}\right], & \text{for } |\mu| \geq \mu_c, \end{cases} \tag{22}$$

where $\mu_c = \cos\alpha_c > 0$ is the loss-cone boundary, and the parameter $\Delta\mu$ determines the sharpness of the loss-cone boundary. The normalization factor A is given by

$$A^{-1} = 2\left[\mu_c + \frac{\sqrt{\pi}}{2}\Delta\mu\,\mathrm{erf}\left(\frac{1-\mu_c}{\Delta\mu}\right)\right], \tag{23}$$

where erf is the error function.

Gaussian beam distribution (GAU; index 4)

Used parameters:

- `Parms[14]` $= 4$.
- `Parms[15]` $= \alpha_0$ is the beam direction, in degrees.
- `Parms[16]` $= \Delta\mu$ is the beam width.

Gaussian beam distribution is given by the expression

$$g_{\mathrm{GAU}}(\mu) = A\exp\left[-\frac{(\mu-\mu_0)^2}{\Delta\mu^2}\right], \tag{24}$$

where $\mu_0 = \cos\alpha_0$ is the beam direction, and the parameter $\Delta\mu$ determines the beam angular width. The above expression represents the beam along the field line for $\alpha_0 = 0$ or $180°$, the transverse beam for $\alpha_0 = 90°$ (coincides with the GLC distribution with $\alpha_c = 90°$), and an oblique beam (or a hollow-beam) otherwise. The normalization factor A is given by

$$A^{-1} = \frac{\sqrt{\pi}}{2}\Delta\mu\left[\mathrm{erf}\left(\frac{1-\mu_0}{\Delta\mu}\right) + \mathrm{erf}\left(\frac{1+\mu_0}{\Delta\mu}\right)\right]. \tag{25}$$

Supergaussian beam distribution (SGA; index 5)

Used parameters:

- `Parms[14]` $= 5$.
- `Parms[15]` $= \alpha_0$ is the beam direction, in degrees.
- `Parms[16]` $= \Delta\mu$ is the beam width.
- `Parms[17]` $= a_4$ is the additional coefficient a_4.

This distribution is very similar to the GAU distribution near its maximum (μ_0) but decreases more rapidly at some angular distance from μ_0. Such a shape is achieved by adding a term with fourth degree of $(\mu - \mu_0)$ to the argument of exponent in (24), that is

$$g_{\rm SGA}(\mu) = A \exp\left[-\frac{(\mu-\mu_0)^2 + a_4(\mu-\mu_0)^4}{\Delta\mu^2}\right], \tag{26}$$

where $\mu_0 = \cos\alpha_0$ is the beam direction, and the beam angular width and shape near the maximum are determined by the parameters $\Delta\mu$ and a_4. The normalization factor A is calculated numerically by using normalization condition (2).

If the angular distribution index differs from the above values $(0-5)$ then the isotropic distribution (index 0) will be used.

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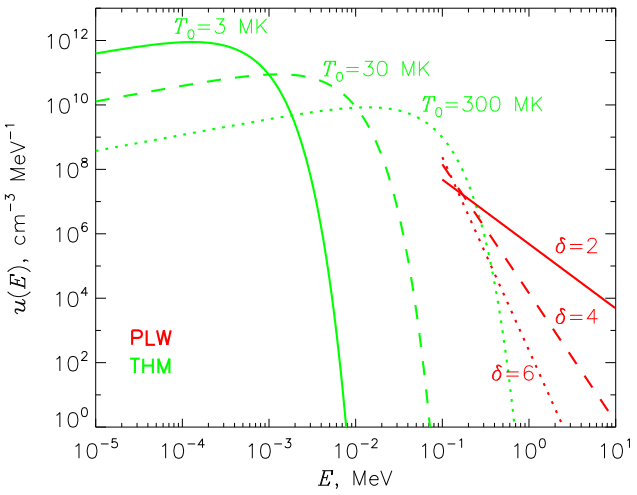

Figure 1: Thermal electron distribution (for $n_0 = 3 \times 10^9$ cm^{-3} and different electron temperatures) and single power-law electron distribution over kinetic energy (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and different power-law indices δ).

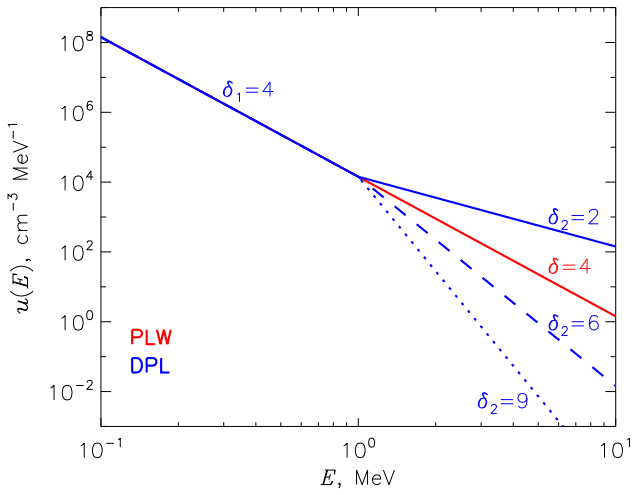

Figure 2: Double power-law electron distribution (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{break} = 1$ MeV, $E_\mathrm{max} = 10$ MeV, $\delta_1 = 4$, and different high-energy power-law indices δ_2). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

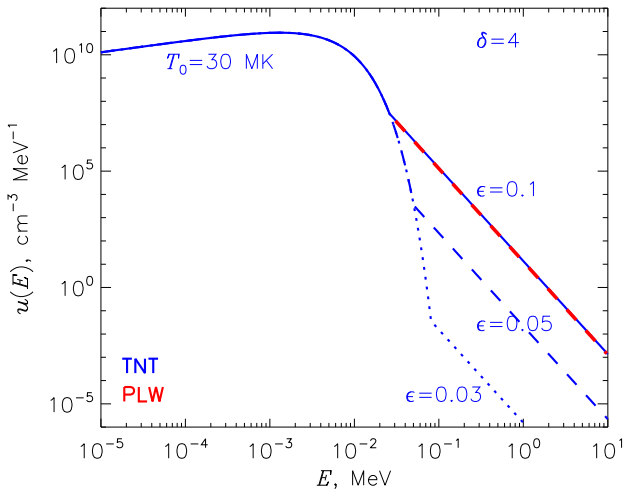

Figure 3: Thermal/nonthermal electron distribution over kinetic energy (for $n_0 = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, $\delta = 4$, and different matching parameters ε). Red dashed line represents the nonthermal "tail" of the thermal/nonthermal distribution; for $\varepsilon = 0.1$, this "tail" behaves as the single power-law distribution with $n_\mathrm{b} = 10^6$ cm^{-3}, $E_\mathrm{min} = 0.03$ MeV, $E_\mathrm{max} = 10$ MeV, and $\delta = 4$.

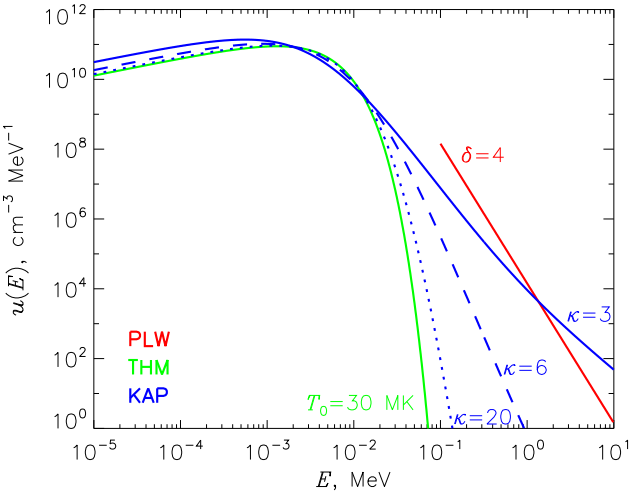

Figure 4: Kappa distribution (for $n_\mathrm{e} = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, and different values of the parameter $\varkappa$). Thermal distribution (for the same particle number density and temperature) and single power-law distribution (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and $\delta = 4$) are given for reference.

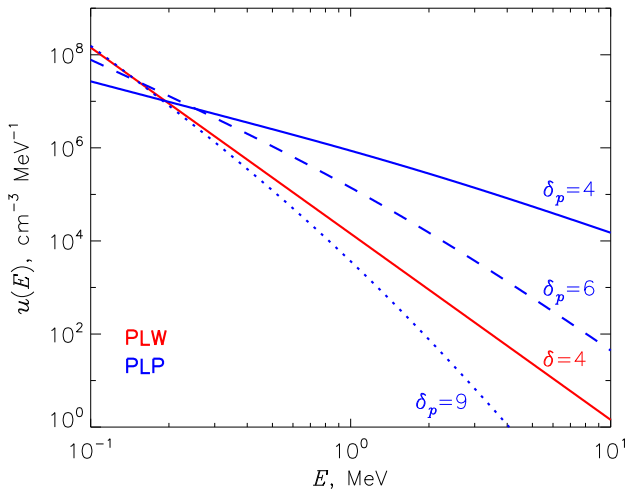

Figure 5: Power-law electron distribution over momentum (for $n_{\rm b} = 3 \times 10^7$ cm^{-3}, $E_{\rm min} = 0.1$ MeV, $E_{\rm max} = 10$ MeV, and different power-law indices δ_p). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

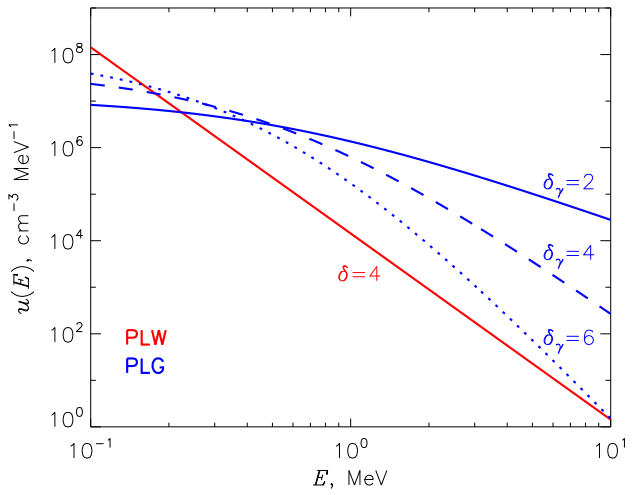

Figure 6: Power-law electron distribution over Lorentz factor (for $n_{\rm b} = 3 \times 10^7$ cm^{-3}, $E_{\rm min} = 0.1$ MeV, $E_{\rm max} = 10$ MeV, and different power-law indices δ_γ). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

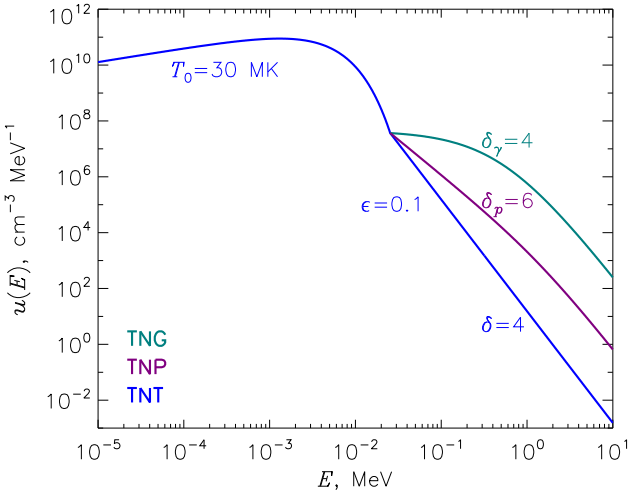

Figure 7: Different thermal/nonthermal electron distributions (for $n_0 = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, $\varepsilon = 0.1$). All the distributions have different numbers of fast electrons above $E_{\rm cr}$.

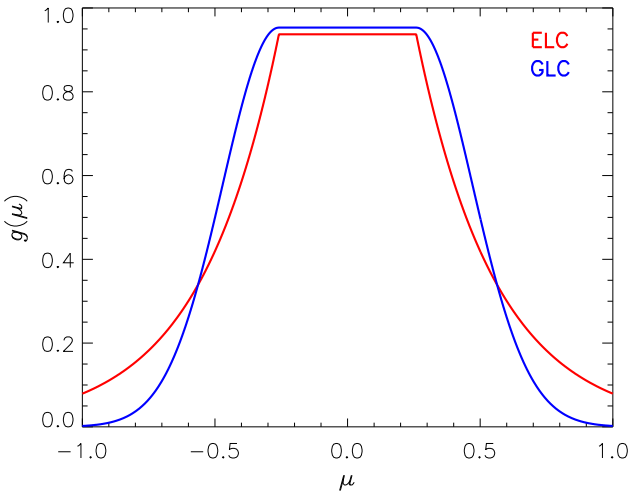

Figure 8: Exponential and gaussian loss-cone distributions (for $\alpha_{\rm c} = 75°$ and $\Delta\mu = 0.3$).

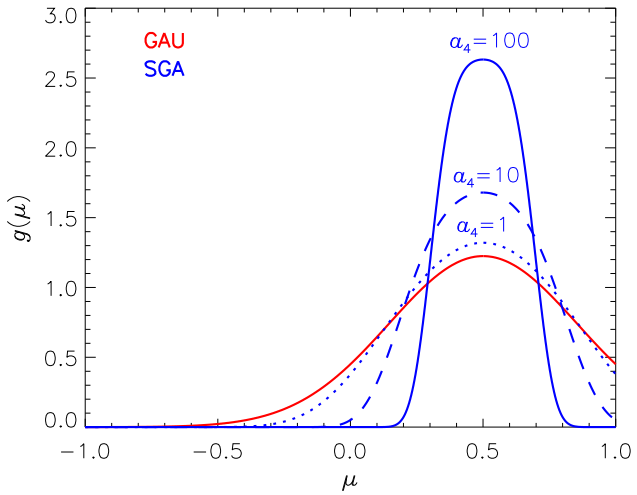

Figure 9: Gaussian distribution (for $\alpha_0 = 60°$ and $\Delta\mu = 0.5$) and supergaussian distribution (for the same α_0 and $\Delta\mu$, and different values of the parameter a_4).