CHIANTI

An Astrophysical Database for Emission Line Spectroscopy

CHIANTI TECHNICAL REPORT No. 2

Computing emission measures with the INTEGRAL_CALC routine

Version 1.0, 28 March 2013, Peter Young

1 Overview

An emission line intensity can be converted into an emission measure value using the CHIANTI routine INTEGRAL_CALC. This report explains how the routine works.

2 Uses

The intensity of an emission line reflects the amount of emitting plasma but it is necessary to account for atomic physics parameters and the element abundance to yield this estimate. The amount of emitting plasma is usually expressed as the *column emission measure* which is $\sim N_{\rm e}^2 h$ where h is the depth of the emitting column and $N_{\rm e}$ is the electron number density. In the case of the solar atmosphere a line intensity represents an integral of the emission from large spatial regions that likely have a range of densities and temperatures and so determining the plasma distribution usually involves solving complex integral equations for multiple emission lines to yield a differential emission measure curve.

Pottasch (1964) came up with a simple method that allows a single line intensity to be directly converted into an emission measure value, and the method described here is a modification of this work. An example of the application of the method is given in Tripathi et al. (2011) where emission measures are derived from multiple emission lines observed by the Hinode/EIS instrument.

3 Method

For historical reasons INTEGRAL_CALC uses different notation for the emission line intensity than that used in the CHIANTI User Guide (CUG), so the two are reconciled here.

Equation 11 of the CUG gives the following expression for the intensity, I, of an optically thin emission line

$$I(\lambda_{ij}) = \int Ab({\rm X}) G(T, \lambda_{ij}, N_{\rm e}) N_{\rm e} N_{\rm H} {\rm d}h \tag{1}$$

where λ_{ij} is the wavelength corresponding to atomic transition i–j, Ab(X) is the abundance of element X relative to hydrogen, G is the contribution function (note that C is used in the CUG), $N_{\rm e}$ and $N_{\rm H}$ are the electron number densities of electrons and hydrogen, and h is the column depth of the plasma. G is given by

$$G(T, \lambda_{ij}, N_{\rm e}) = \frac{h\nu_{ij}}{4\pi} \frac{A_{ij}}{N_{\rm e}} \frac{N_j({\rm X}^{+m})}{N({\rm X}^{+m})} \frac{N({\rm X}^{+m})}{N({\rm X})}. \tag{2}$$

where $h\nu_{ij} = \Delta E$ is the energy for the transition, A_{ij} is the radiative decay rate, $N_j({\rm X}^{+m})$ is the population of the emitting level j of the ion ${\rm X}^{+m}$. We can re-write this as

$$G_\lambda(T, N_{\rm e}) = \frac{\Delta E}{4\pi} \frac{n_j A_{ij}}{N_{\rm e}} F(T) \tag{3}$$

where n_j is the population of level relative to the population of the emitting ion as a whole and $F_m(T)$ is the ionization fraction of ion ${\rm X}^{+m}$.

We define an alternative version of the contribution function as

$$\mathscr{G}_\lambda(T, N_\mathrm{e}) = \frac{4\pi}{\Delta E}\frac{N_\mathrm{H}}{N_\mathrm{e}} G_\lambda(T, N_\mathrm{e}) \tag{4}$$

(note that $N_\mathrm{H}/N_\mathrm{e}$ is approximately a constant with value around 0.85 for solar plasmas with temperatures $\geq 10^5$ K) and so Eq. 1 becomes

$$4\pi I_\lambda = \Delta E A b(\mathrm{X}) \int \mathscr{G}_\lambda(T, N_\mathrm{e}) N_\mathrm{e}^2 \mathrm{d}h. \tag{5}$$

Fundamental approximations behind INTEGRAL_CALC are to assume the density is a known constant in the region the line is formed, and that $\mathscr{G}$ is constant in the temperature region where $\mathscr{G}$ has its peak value, and zero outside of this region. Formally, we define

$$\mathscr{G}^*_\lambda(T) = \begin{cases} C_\lambda & |\log T - \log T_\mathrm{mem}| < 0.15 \\ 0 & |\log T - \log T_\mathrm{mem}| > 0.15 \end{cases} \tag{6}$$

for some constant C_λ, where T_mem is the temperature at which $\mathscr{G}$ has its maximum value. We require that

$$\int \mathscr{G}_\lambda(T) dT = \int \mathscr{G}^*_\lambda(T) dT \tag{7}$$

and so C_λ is given by

$$C_\lambda = \frac{\int \mathscr{G}_\lambda(T) dT}{T_\mathrm{mem}(10^{0.15} - 10^{-0.15})}. \tag{8}$$

Eq. 5 is then written as

$$4\pi I_\lambda = \Delta E A b(\mathrm{X})\, C_\lambda \int N_\mathrm{e}^2 \mathrm{d}h \tag{9}$$

$$= \Delta E A b(\mathrm{X})\, C_\lambda\, \mathrm{EM}(h) \tag{10}$$

where $\mathrm{EM}(h)$ is the column emission measure, thus the emission measure can be directly related to the observed line intensity. If N_e is known from a density diagnostic then the column depth can be estimated from $h = \mathrm{EM}(h)/N_\mathrm{e}^2$.

4 Implementation

The key part of the routine is to compute the integral which is done by computing the function

$$f_i = \ln\, 10\, \varepsilon_{ij}(T_i, N_\mathrm{e}) F(T_i) T_i \tag{11}$$

where ε is the line emissivity computed by the CHIANTI routine EMISS_CALC, F is the ionization fraction, T_i the temperature and N_e the electron density.

The following integral is then computed:

$$\int f \mathrm{d}\,(\log\, T) = \sum f_i\, d(\log\, T_i) \tag{12}$$

using the IDL routine INT_TABULATED, with $\log\, T_i$ specified at 0.01 dex intervals.

We then have

$$C_\lambda = \frac{N_\mathrm{e}}{0.7046\,T_\mathrm{mem}} \frac{N_\mathrm{H}}{N_\mathrm{e}} \sum f_i\, d(\log\, T_i) \tag{13}$$

where T_mem is the temperature at which the function $\varepsilon\, F(T)$ has its peak. The proton-to-electron ratio is computed using the IDL routine PROTON_DENS, which makes use of the default CHIANTI element abundance file (given by system variable `!abund_file`). An alternative file can be specified with the keyword `abund_file` to INTEGRAL_CALC.

5 Outputs

The routine gives the following outputs: T_mem, C_λ, $\Delta E\, C_\lambda$ and $4\pi/(\Delta E\, C_\lambda)$ which are printed to the screen. Alternatively they can be returned to IDL through the optional output OUTSTR=. Examples of the output from INTEGRAL_CALC are given in Appendix B.

6 Types of call

The most simple call is, e.g.,

```
IDL> integral_calc,'fe_13'
```

which will display results for the 10 strongest lines. Note that user input is required to blend lines (if necessary) or plot the emissivity functions.

Results for a specific wavelength range can be printed with:

```
IDL> integral_calc,'fe_13',wrange=[200,205]
IDL> integral_calc,'fe_13',wrange=[200,205],/all
```

The first call will take the five strongest lines in the specified wavelength range, while the second prints all lines in the wavelength range. An example of the output from the routine is given in Appendix B.

If INTEGRAL_CALC is to be called from within another routine then it is necessary to directly specify a specific transition. This can be done with:

```
IDL> integral_calc,'fe_13',index=39127,outstr=outstr
```

The index is the index of the transition within the emissivity structure produced by EMISS_CALC. In the example above the strong λ202.04 line is selected.

7 The /QUICK keyword

The ionization fractions are tabulated at 0.05 dex intervals in log T, and INTEGRAL_CALC computes the emissivities, ε, at 0.1 dex intervals as they are more slowly varying in temperature. Both quantities are interpolated onto 0.01 dex intervals for the integration.

The call to EMISS_CALC can be very slow for large ion models such as the coronal iron ions. For this reason a keyword /QUICK is available that speeds up the calculation by:

1. Only using temperatures for which the ionization fraction is $\geq F_{\max}/1000$, where $F_{\max}$ is the maximum of the ionization fraction.
2. Only computing the emissivities for a reduced number of temperatures (the present implementation uses 6 temperatures) that span the ionization fraction temperatures.

This approximation is found to work well for the coronal iron ions, giving results within 1% of the original implementation. There are examples where /QUICK does not work well, for example H I and O VI, where the ionization fraction extends over a wide temperature range. The code has an in-built check on whether the approximation is working well and a warning is printed if not.

References

Pottasch, S. R. 1964, Space Sci. Rev., 3, 816

Tripathi, D., Klimchuk, J. A., & Mason, H. E. 2011, ApJ, 740, 111

A Document history

Version 1.1, 6-Feb-2017. Revised Sect. 3 to reconcile with CUG; added Appendix.

B Example output

The following output was generated using CHIANTI 8. The output parameters should be related to Eq. 10. "Emiss ind" is the index of the transition with the structure returned by the routine EMISS_CALC.

```
IDL> integral_calc,'fe_13',wrange=[200,205]

Ion balance file: /Users/young/ssw/packages/chianti/dbase/ioneq/chianti.ioneq
Log (density/cm^-3): 10.0

      Emiss ind    Lambda    T_mem   C_lambda  DE*C_lambda 4pi/DE*C_lambda

  1       26183     200.021    6.25  1.0352e-10  1.0278e-20  1.2226e+21
  2       26381     201.126    6.25  1.1015e-10  1.0877e-20  1.1553e+21
  3       26520     202.044    6.25  1.4385e-10  1.4140e-20  8.8869e+20
  4       26842     203.795    6.25  1.3829e-10  1.3477e-20  9.3244e+20
  5       26851     203.826    6.25  3.8453e-10  3.7468e-20  3.3539e+20
```